Dr. Armen V. Papazian, King's College Cambridge, January 2024

OUR CURRENT FINANCIAL MATHEMATICS

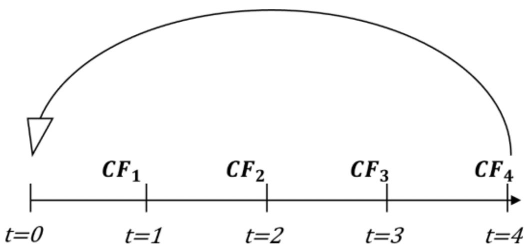
RISK & TIME Without SPACE

Our current financial mathematics is derived from a value framework built on two principles of value: *Risk and Return* and *Time Value of Money*. This reveals an entire corpus of equations built on two primary axes of analysis, *Risk* and *Time*, without any reference to or use of context parameters. Our financial mathematics is built in a *risktime* universe without context, missing the analytical dimension of *Space* - our physical context.

Stakeholder	*Risk*	*Time*
Mortal Risk-Averse Return-maximising Investor	***Risk and Return***: The higher the risk the higher the expected return— given the risk-averse nature of investors, higher risks imply higher expectations of reward.	***Time Value of Money***: A dollar ($1) today is worth more than a dollar ($1) tomorrow— because a dollar today can earn interest/return by tomorrow and be more than a dollar by tomorrow.

DISCOUNTING

The analytical dimensions and principles of value built on Risk and Time translate into equations where the discounting of cash flows is a primary tool to measure the risk and time value of cash flows.



Sample Bond Valuation Equations

$$Bond\ Price = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} + \frac{P}{(1+r)^n}$$

$$Bond\ Price = \sum_{t=1}^{n\times m} \frac{CF_t}{\left(1+\left(\frac{r}{m}\right)\right)^t} + \frac{P}{\left(1+\left(\frac{r}{m}\right)\right)^{n\times m}}$$

$$Bond\ Price = \left(\frac{C}{m}\right) \times \left(\frac{1-\left(\frac{1}{\left(1+\frac{r}{m}\right)^{n\times m}}\right)}{\left(\frac{r}{m}\right)}\right) + \frac{P}{\left(1+\frac{r}{m}\right)^{n\times m}}$$

Sample of Stock and Firm Valuation Equations

$$P_0 = \frac{D_1}{r-g}$$

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

$$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+WACC)^t} + \frac{P_n}{(1+WACC)^n}$$

$$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+WACC)^t} + \frac{D_{n+1}}{(WACC-g).(1+WACC)^n}$$

$$Firm\ Value = \sum_{t=1}^{n} \frac{FCFF_t}{(1+WACC)^t} + \frac{FCFF_{n+1}}{(WACC-g).(1+WACC)^n}$$

NET PRESENT VALUE CASH FLOW VALUATION MODEL

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} \qquad NPV = CF_0 + \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}$$

$$\boldsymbol{N}et\ Present\ Value = \boxed{-\ II} + \boxed{\sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}}$$

WITHOUT SPACE & DISCOUNTING THE NON-ACTUAL

In the above Net Present Value equation the abstraction of context/space happens when the treatment of the Initial Investment (II) is limited to ascribing it a negative sign (-) as an outflow for the mortal risk-averse return-maximising investor, without any further consideration of or investigation into the impact(s) of the investment. **The model omits the space impact it would take to achieve or expect the future cash flows being discounted to the present.**

Moreover, the relative mathematical sophistication of the equation is focused on the non-actual element of the equation, CF_t in the future, , while the only actual element in the equation, the Initial Investment (II), , is treated only with a negative sign (-). This bias towards the non-actual future expected cash flows, coupled with the abstraction of space impact, reveals the necessity to quantify and account for the space impact of cash flows and compound them into the future when relevant.

NON-DISCOUNTING MODELS

Not all of our value and return models in finance make use of discounting. However, they are still based on *risk* and *time*, and do not incorporate any context parameters – Space remains missing as an analytical dimension and our physical context. Moreover, in many of our models, such as the CAPM, risk itself is measured as the relative time-based performance of returns.

Sample of Asset Pricing Models: CAPM and 3FM

$$R_i = R_f + \beta_i \times (R_m - R_f) \qquad Beta_i = \beta_i = \frac{Covariance_{R_i,R_m}}{Variance_{R_m}}$$

$$E(R_i) - R_f = b_1\big(E(R_M) - R_f\big) + s_i E(SMB) + h_i E(HML)$$

Modigliani Miller Corporate Value and Capital Structure Model

$$V_j = \left(S_j + D_j\right) = \frac{\overline{X}_j}{\rho_k}$$

$$i_j = \rho_k + (\rho_k - r)\frac{D_j}{S_j}$$

Black and Scholes Option Pricing Model

$$C = SN(d) - Le^{-rt}N(d - \sigma\sqrt{t})$$

$$d = \frac{\ln\frac{S}{L} + \left(r + \frac{\sigma^2}{2}\right)t}{\sqrt[\sigma]{t}}$$

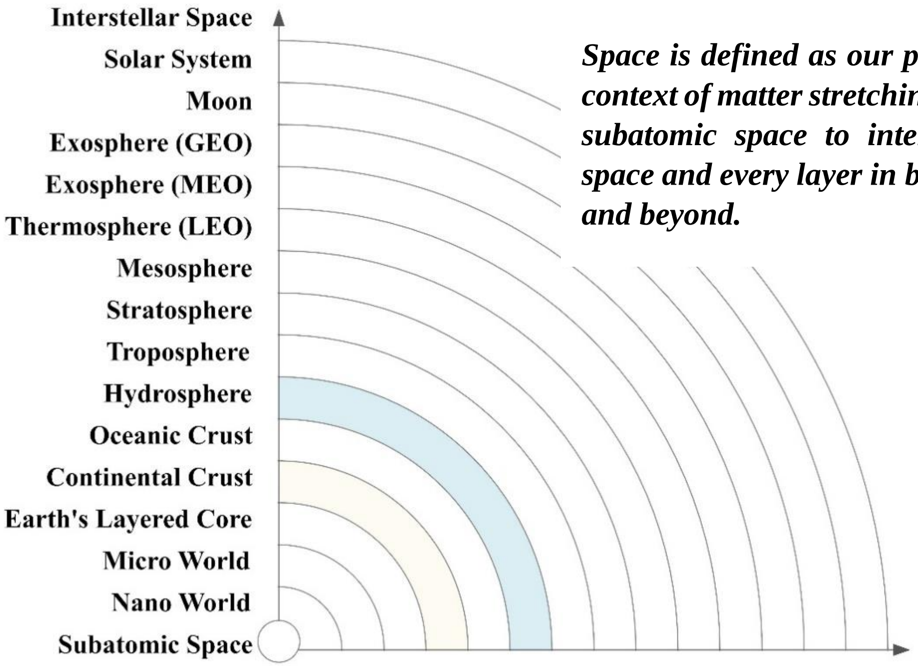
See Brealey, Myers, and Allen (2020), Pike, Neale, Akbar, and Linsley (2018), Watson and Head (2016), Fama and French (1996, 2004, 2015), Gordon (1959), Gordon and Gordon (1997), Gordon and Shapiro (1956), Modigliani and Miller (1958), Ross (1976), Roll and Ross (1980), Sharpe (1964), Lintner (1965), Merton, (1973), Black and Scholes (1973), Nobel Prize (1997) and others. See Papazian (2022) for a detailed discussion of the above, and the absence of space and space impact.

SPACE-ADJUSTED FINANCIAL MATHEMATICS

SPACE & RISK & TIME

A space-adjusted financial mathematics begins by the introduction of *Space* into our value framework, as analytical dimension, and our physical context - a context that stretches from subatomic space to interstellar space and every layer in between and beyond. This reveals the necessity to introduce a third core principle of value into our framework, the *Space Value of Money*, which defines our relationship with *Space*, just like *Risk and Return* and *Time Value of Money* define our relationship with *Risk* and *Time*.

Space is defined as our physical context of matter stretching from subatomic space to interstellar space and every layer in between and beyond.



THE SPACE VALUE OF MONEY

The space value of money principle complements time value of money and risk and return. **It establishes our spatial responsibility and requires that a dollar ($1) invested in space has, at the very least, a dollar's ($1) worth of positive impact on space.**

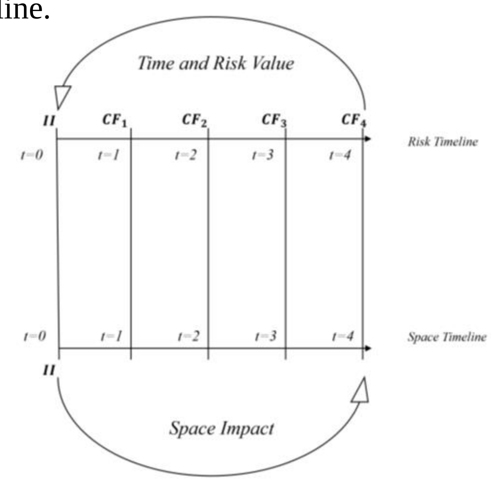
The space value of money principle acts as the bottom threshold of investment acceptability because it requires that investments have a positive impact on space taking into account all the layers of space affected by the investment.

THE DOUBLE TIMELINE & NET SPACE VALUE (NSV)

To account for and measure the space impact of cash flows across all affected space layers we start with the double timeline.

WHY SPACE LAYERS? $\sum_{s=1}^{S}$

a) Value Chain
b) Impact Intensity
c) Clean-up Technology
d) Costs



$$\textbf{Gross Space Value}_{T,S} = \textbf{NSV} + \textbf{II}$$

$$NSV + II = GSV \mid NSV = 0$$

$$\textbf{Space Neutral Position} \mid \textbf{NSV} = \textbf{0}$$

$$\textbf{GSV} = \textbf{II} \mid \textbf{Minimum Space Value Condition}$$

$$NSV_{T\&S} = \{\textbf{Planetary, Human, and Economic Impact}\}_{\textbf{All S Layers \& T Periods}}$$

$$T = Total\ Number\ of\ Years\ of\ Investment\ being\ Considered$$

$$S = All\ Space\ layers\ Involved\ and/or\ affected\ by\ the\ Investment$$

$$NSV_{T\&S} = \sum_{t=1}^{T}\sum_{s=1}^{S} Pollution\ \&\ Biodiversity\ Impact$$
$$+ \sum_{t=1}^{T}\sum_{s=1}^{S} Human\ Capital\ \&\ R\ and\ D\ Impact$$
$$+ \sum_{t=1}^{T}\sum_{s=1}^{S} New\ Asset\ \&\ New\ Money\ Impact$$

Impact Aspect		Net Space Value	$g \times (PI_{T,S,P} + BI_{T,S,B} + HCI_{T,S} + RDI_{T,S,N} + NAI_{D,S,A} + NMI_{T})$
PLANETARY		Pollution Impact	$PI_{T,S,P} = \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{p=1}^{P} Q_{pst} \times C_{pst}$
		Biodiversity Impact	$BI_{T,S,B} = \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{b=1}^{B} A_{bst} \times R_{bst}$
HUMAN		Human Capital Impact	$HCI_{T,S} = f \times \sum_{t=1}^{T}\sum_{s=1}^{S} E_{st} + T_{st} + H_{st} + I_{st} + C_{st} + S_{st}$
		R and D Impact	$RDI_{T,S,N} = \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{n=1}^{N} h_n \times RD_{tsn}$
ECONOMIC		New Asset Impact	$NAI_{D,S,A} = \sum_{s=1}^{S}\sum_{a=1}^{A} k_a \times BVA_{asD}$
		New Money Impact	$NMI_T = (II \times DR \times BLR) + mm \times (II + X_T - M_T)$

SPACE GROWTH RATE

$$SPR = \sqrt[T]{\frac{NSV_{T,S}}{II}} - 1$$

NEGATIVE SPACE IMPACT ADJUSTED NPV

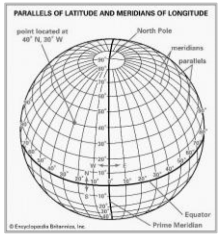
$$NIA\ NPV = -\left|NNSV_{T,S}\right| - II + \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}$$

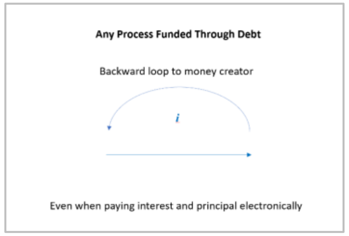
IMPLICATIONS FOR MONEY MECHANICS

If investors must respect the space value of money principle, and earn their returns with a positive space impact, then money creators, whether they are commercial or central banks, must also follow the same principle. The hardwiring of sustainability into financial mathematics will naturally require a comprehensive reassessment of instruments and investments used by money creators.

Three systemic implications for debt-based money in all its forms:

Currency, Deposits, and Central Bank Reserves = IOUs= Debt-based





Calendar Time: Given the fixed pace of the artificially projected construct, i.e., calendar time, a money mechanics based on instruments linked to calendar time obligations acts as a muzzle limiting our ability to invest in space timelessly.

Monetary Gravity: Given the backward loop to the money creator, calendar time linked debt-based money imposes a limit on the distance a process/investment can go in space before it must return to pay interest to some bank, acting as an unnecessary leash in space.

Monetary Hunger: Given that money is continuously created through debt, irrespective of capital accumulation, debt-based money creates monetary hunger in any economy, and given the actual threat of default, it acts as a whip and triggers unsustainable practices as borrowers choose to serve their debts before the environment.

Papazian, Armen. 2022. *The Space Value of Money: Rethinking Finance Beyond Risk and Time*. New York: Palgrave Macmillan. https://doi.org/10.1057/ 978-1-137-59489-1

Papazian, Armen. 2023. *Hardwiring Sustainability into Financial Mathematics: Implications for Money Mechanics*. New York: Palgrave Macmillan. https://doi.org/10.1007/978-3-031-45689-3